FORM TP 2016142

TEST CODE 02205020

MAY/JUNE 2016

CARIBBEAN EXAMINATIONS COUNCIL

CARIBBEAN ADVANCED PROFICIENCY EXAMINATION®

APPLIED MATHEMATICS

MATHEMATICAL APPLICATIONS

UNIT 2 – Paper 02

2 hours 30 minutes

READ THE FOLLOWING INSTRUCTIONS CAREFULLY.

1. This paper consists of THREE sections. Answer ALL questions from the THREE sections.
2. Write your answers in the spaces provided in this booklet.
3. Do NOT write in the margins.
4. Unless otherwise stated in the question, all numerical answers MUST be given exactly OR to three significant figures as appropriate.
5. If you need to rewrite any answer and there is not enough space to do so on the original page, you must use the extra page(s) provided at the back of this booklet. **Remember to draw a line through your original answer**.
6. **If you use the extra page(s) you MUST write the question number clearly in the box provided at the top of the extra page(s) and, where relevant, include the question part beside the answer.**

Examination Materials:

Mathematical formulae and tables (Revised 2010)
Electronic calculator
Ruler and graph paper

DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO.

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Sequential Bar Code

SECTION A

MODULE 1: DISCRETE MATHEMATICS

Answer BOTH questions.

1. (a) Show that the Boolean expression $(p \wedge \sim q) \vee (\sim p \wedge q)$ is equivalent to $(p \vee q) \wedge (\sim p \vee \sim q)$ using

(i) truth tables

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[4 marks]

(ii) the laws of Boolean algebra.

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[4 marks]

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(b) Draw the switching circuits for

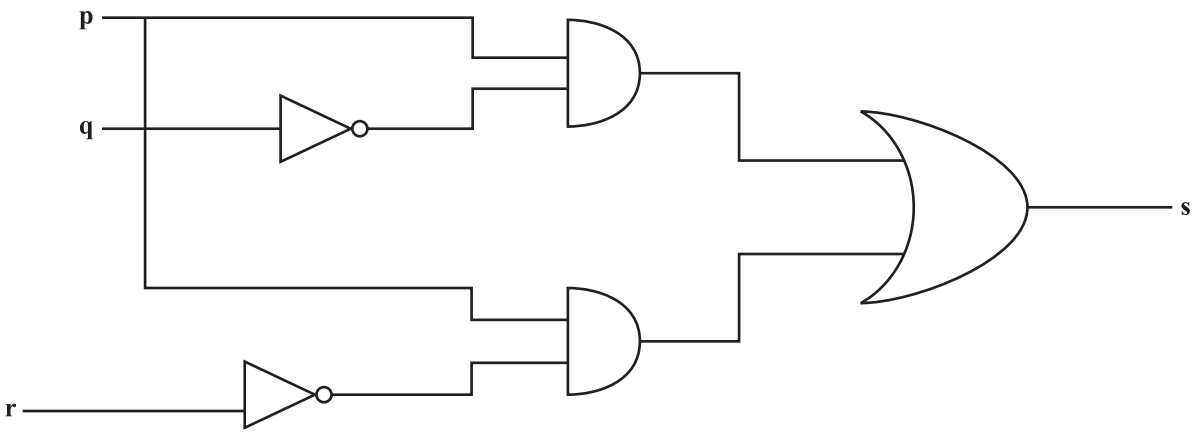
(i) $(p \wedge \sim q) \vee (\sim p \wedge q)$.

[3 marks]

(ii) $(p \vee q) \wedge (\sim p \vee \sim q)$.

[3 marks]

(c) The following diagram represents the logic circuit with inputs at **p**, **q** and **r** and output **s**.



Determine the Boolean expression for **s**.

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[3 marks]

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(d) Let p, q, r and s be the propositions:

p: A person who eats red meat

q: A person who has a high cholesterol reading

r: A person with a normal cholesterol reading

s: A person who suffers a heart attack

Using the propositions p, q, r and s and the connectives $\vee$, $\wedge$, $\rightarrow$ and $\sim$, write down an expression for EACH of the following statements.

(i) If a person eats red meat then that person may have a high cholesterol reading or suffer a heart attack.

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[2 marks]

(ii) If a person does not suffer a heart attack then that person has a normal cholesterol reading and does not eat red meat.

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[2 marks]

(iii) Construct a truth table for (d) (i) and state with reason whether it is a tautology.

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[4 marks]

Total 25 marks

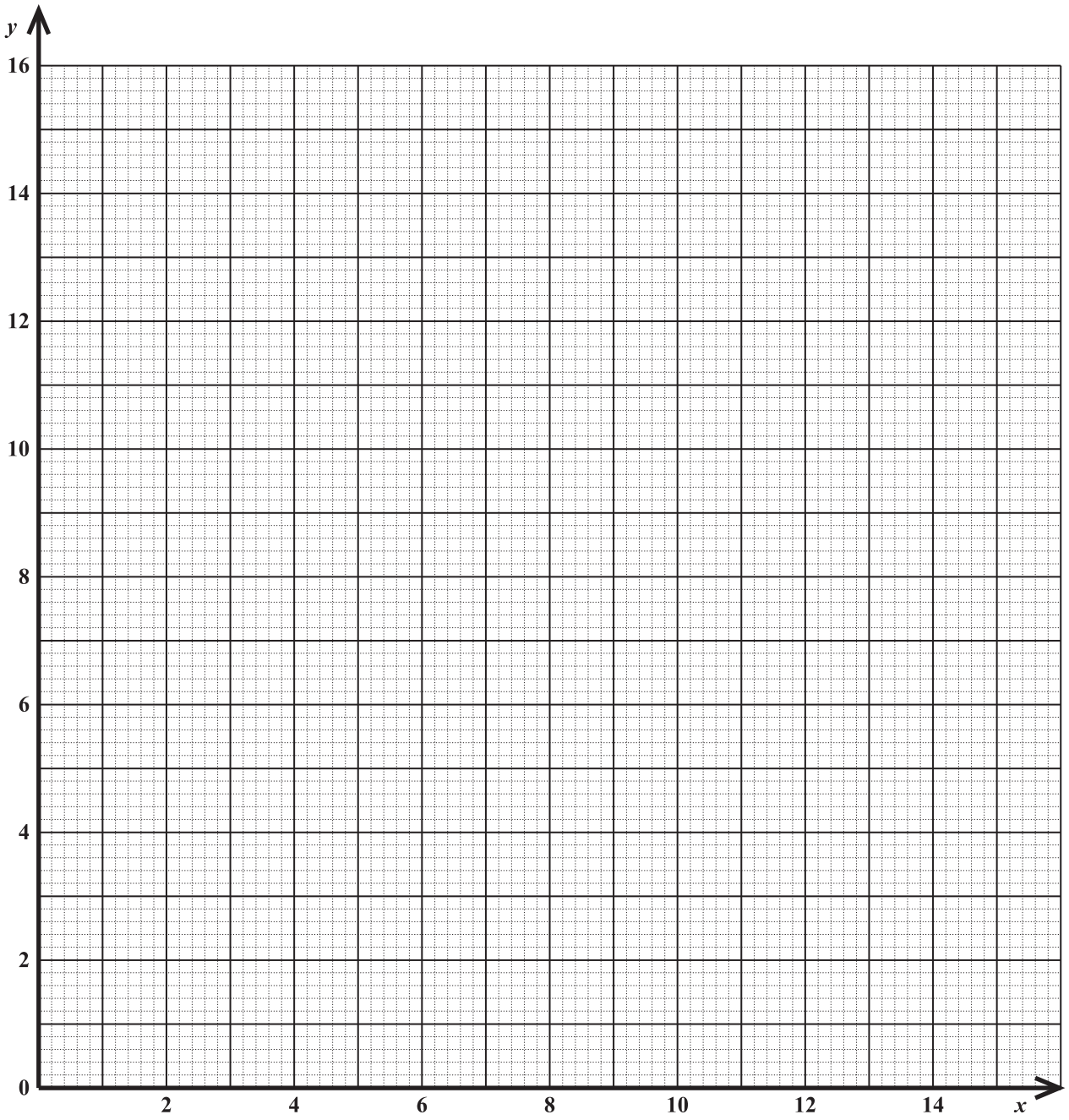
2. (a) A linear programming model is represented by the following:

Maximize: $P = 6x + 4y$
subject to: $4x + 6y \leq 48$
$4x + 2y \leq 32$
$0 \leq y \leq 5$
$x \geq 0$.

(i) On the grid provided **on page 9**, draw the system of inequalities. **[9 marks]**

(ii) Using the same grid, shade the feasible region. **[2 marks]**

(iii) Use your diagram to find the maximum value of P. **[3 marks]**



(b) The times, in minutes, taken by four drivers, A, B, C and D, to take passengers to the towns R, H, S and P are given in the following table.

	R	***H***	***S***	***P***
A	23	31	25	28
B	30	20	33	29
C	20	25	28	25
D	35	19	20	30

(i) Use the Hungarian algorithm to determine the town to which EACH driver should be assigned in order to minimize the travel time.

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[9 marks]

(ii) Determine the total time taken by the four drivers.

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[2 marks]

Total 25 marks

SECTION B

MODULE 2: PROBABILITY AND DISTRIBUTIONS

Answer BOTH questions.

3. (a) A student must present a portfolio with 12 paintings for examination in Fine Arts. There are 10 water colour paintings and 7 oil paintings from which a selection can be made to create a portfolio.

(i) How many portfolios can be created?

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[3 marks]

(ii) Determine the number of portfolios with 8 water colours and 4 oil paintings that can be created.

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[4 marks]

(iii) Hence, calculate the probability that 8 water colours and 4 oil paintings are selected for the portfolio.

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[2 marks]

(b) To start a game, a player must get a six on the toss of a fair die.

(i) Calculate the probability that the player is able to start the game on the third toss of the die.

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[4 marks]

(ii) Calculate the probability that **at most** FIVE tosses are necessary to start the game.

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[3 marks]

(iii) Determine the expected number of tosses a player must make to start the game.

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[2 marks]

(c) On average 2.5 faulty reports are made to a company's switchboard per day. Calculate the probability that **exactly**

(i) FOUR faulty reports will be made on Monday

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[3 marks]

(ii) FIVE faulty reports will be made over the next five-day period.

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[4 marks]

Total 25 marks

4. (a) A researcher wants to determine whether there is an equal distribution of grades for a number of students who write a public examination. The following table shows the frequencies of the grades awarded to a sample of 100 students who took the examination.

Grade	A	B	C	D	F
Observed Frequency	14	22	30	18	16

A chi squared, χ^2, test at the 5% level of significance is used to examine this concern.

(i) Clearly state appropriate null and alternative hypotheses for the test.

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[2 marks]

(ii) Determine the critical region for the test.

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[3 marks]

(iii) Calculate the value of the test statistic.

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[4 marks]

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(iv) State, with reason, a valid conclusion for the test.

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[2 marks]

(b) A continuous random variable has the probability density function, f, given by

$$f(x) = \begin{cases} k\,(4 - x), & 0 \leq x \leq 3 \\ 0, & \textit{otherwise} \end{cases}$$

(i) Determine the value of k.

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[3 marks]

(ii) Calculate $P(X > 1)$.

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[3 marks]

(iii) Calculate $E(X)$.

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[3 marks]

(iv) State fully the cumulative distribution function $F(x)$.

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[3 marks]

(v) Hence, or otherwise, find $P(1.5 < X < 2)$.

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[2 marks]

Total 25 marks

SECTION C

MODULE 3: PARTICLE MECHANICS

Answer BOTH questions.

[Take g as 10 ms^{-2}]

5. (a) A fixture is attached to two strings A and B, which are at $\theta°$ and 30° respectively to a horizontal ceiling. String A, which is attached at $\theta°$, has a tension of T_A = 50 N. Use Lami's theorem, or otherwise, to show that

(i) the weight of the light fixture is $50\left(\frac{\cos\theta}{\sqrt{3}} + \sin\theta\right)$

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[9 marks]

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(ii) the tension of string B, $T_B = \frac{100}{\sqrt{3}} \cos \theta$.

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[3 marks]

(b) A wrecker of mass 1050 kg pulls a train of buses of mass 3550 kg down an incline plane of gradient 1 in 5 ($\sin\theta = \frac{1}{5}$) against a track resistance of 250 N. The speed is reduced from 80 km h^{-1} to 44 km h^{-1} with the engine shut off and the braking force 10 kN.

(i) Draw a clearly labelled diagram to illustrate the information above after the engine is shut off.

[3 marks]

(ii) Determine the decelerating force.

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[4 marks]

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(iii) Determine the time taken to reach a speed of 44 km h^{-1}.

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[6 marks]

Total 25 marks

6. (a) A body of mass 15 kg is pulled at a constant speed of 4 ms^{-1} up a plane inclined at 30° to the horizontal by a force parallel to the plane. If the coefficient of friction is 0.2, calculate

(i) the tractive force acting parallel to the plane

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[5 marks]

(ii) the work done, to the **nearest whole number**, after one second to move the body parallel to the plane

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[3 marks]

(iii) the power developed during that time.

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[1 mark]

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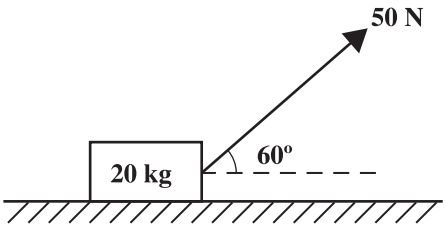
(b) The following diagram shows a particle of mass 20 kg being pulled by a force 50 N (inclined at 60° to the horizontal) along a rough horizontal surface.



If the particle is just about to slip, find the coefficient of friction of the surface.

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[6 marks]

(c) A vehicle, *A*, of mass 3000 kg travelling at 2 m s^{-1} collides with another vehicle, *B*, of mass 5 000 kg travelling at 1 m s^{-1} in the opposite direction. After collision, vehicle *A* moves in the same direction with a speed of 1.5 m s^{-1} and is brought to rest by a frictional force of 150 N.

(i) Calculate the speed of vehicle *B* after impact.

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[4 marks]

(ii) State the direction of vehicle *B* after collision.

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[1 mark]

(iii) Calculate the time for vehicle *A* to come to rest after the collision.

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[5 marks]

Total 25 marks

END OF TEST

IF YOU FINISH BEFORE TIME IS CALLED, CHECK YOUR WORK ON THIS TEST.